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# STABILITY ANALYSIS OF EQUALITY POINT MATHEMATICS MODEL OF INFLUENZA VIRUS IN THE HUMAN BODY WITH HERBAL TREATMENT THERAPY

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***Abstract.*** *This study discusses a mathematical model for the transmission of influenza virus in the human body. The mathematical model used is the SITR model. As an effort to inhibit the influenza virus, the model is considered as herbal treatment therapy.*
*The purpose of this study was to analyze the stability of the equilibrium point of the mathematical model of the influenza virus in the human body with herbal medicine therapy. The method used in analyzing the problem is literature study. The steps taken are determining the problem, literature study, analysis and problem solving and drawing conclusions. As a result of the research, the model obtained is. From this model, there are two equilibrium points, namely the disease-free equilibrium point and the endemic equilibrium point. The analysis carried out resulted in the basic reproduction ratio number. After analyzing the two equilibrium points, it can be concluded that the disease-free equilibrium point will be locally asymptotically stable . Meanwhile, the endemic equilibrium point will be locally asymptotically stable . Furthermore, to illustrate the model, a model simulation is carried out using the Maple.*

***Keywords:*** *Mathematical Model, Herbal Medicine, Equilibrium Point, Influenza Virus*

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## INTRODUCTION

A mathematical model is a set of equations or inequalities that express the behavior of a real problem. The mathematical model that has been formed will be analyzed so that the model made is representative of the problems discussed. Many problems that arise from various fields of health, chemistry, biology and others that can be made a mathematical model . Mathematical models to analyze the spread of disease include the SIR (Susceptible - Infected - Recovered), SEIR (Susceptible - Exposed - Infected - Recovered) epidemic model, and others (Hazarika, 2016).

Influenza is a respiratory tract infection caused by the influenza virus. Clinical symptoms that occur ranging from mild to severe infection and can even lead to complications and death. In fact, influenza is also often experienced by the people of Indonesia and is often known as flu. Influenza that occurs in Indonesia (other than bird flu)

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generally gives mild to moderate symptoms. Meanwhile, in countries with four seasons such as America, influenza infection known as Seasonal Influenza often results in severe infections (Gallo et al., 2020).

In an effort to prevent and treat Influenza A, the administration of antiviral drugs and vaccinations is carried out. Although vaccination is the best option to protect against influenza virus infection, controlling influenza A virus infection using antivirals is starting to experience problems with the emergence of new strains that are resistant to existing antivirals. One alternative to overcome this problem is to use herbal antivirals(Nanda et al., 2022).

Influenza has a fairly high mortality rate. Therefore, a mathematical model is needed to overcome the problem of the spread of influenza. Naba Kumar and Shanmukha(2017) have developed a model of the spread of influenza which is defined as a system of nonlinear differential equations. The mathematical model of the spread of influenza that will be discussed in this study is the SITR epidemic model. The SITR Susceptible-Infected-Treatment-Recovered epidemic model is a disease spread model that divides the population into four groups, namely a group of individuals who are healthy but can be infected with a disease (Susceptible), a group of individuals who are infected with the disease and can recover from the disease (infected), groups of individuals undergoing treatment (Treatment) and groups of individuals who have recovered and are immune to disease (Recovered). The SITR epidemic model is a development of the classic SIR model. The SIR epidemic model assumes that an infected individual will recover while the SITR model represents a situation when an infected individual must undergo treatment to recover (Khanh, 2016).

Therefore, the authors developed a mathematical model of influenza disease in the human body with herbal treatment therapy because several types of herbs have benefits for overcoming the initial symptoms of influenza in the human body infected with the influenza virus. From the model that has been formed, it is necessary to analyze the stability of the model. By first finding the equilibrium points of the SITR model, then determining the basic reproduction number $R_0$, forming a Jacobian matrix, forming a characteristic polynomial that forms the eigenvalues. From the eigenvalues, it can then be known whether or not each equilibrium point is stable (Hauferson, 2017).

Based on the formulation of the problem above, the purpose of writing this thesis is to develop a mathematical model of the influenza virus to analyze the stability of the equilibrium point of the mathematical model of the influenza virus in the human body with herbal medicine therapy and then simulate it.

## METHOD

### System of Differential Equations

Given a differential equation of the form

$$\dot{x} = f(t, x) \qquad \text{………(1)}$$

where $\dot{x} = \frac{dx}{dt}$ express the derivative of x with respect to t. Equation (1) is called a non-autonomous equation because there is a variable free t that appears explicitly. In general, a system of first-order linear differential equations with a variable the dependent variable $x_1, x_2, \ldots, x_n$ and the independent variable t, are expressed as follows:

$$\frac{dx_1}{dt} = f_1 \quad (x_1, x_2, \dots, x_n)$$
$$\frac{dx_1}{dt} = f_1 \quad (x_1, x_2, \dots, x_n)$$
$$\dots$$
$$\frac{dx_1}{dt} = f_1 \quad (x_1, x_2, \dots, x_n)$$
………(2)

Nonlinear differential equations are ordinary differential equations that are not linear (Rosyada et al., 2019). A differential equation is said to be nonlinear if it satisfies at least one of the following criteria

1. Contains the dependent variable from its derivatives to powers other than one.
2. There is a multiplication of the dependent variable and/or its derivatives.
3. There is a transcendental function of the dependent variable and its derivatives

**Equilibrium Point;** Equilibrium point (equilibrium point) is a fixed point that does not change against time. Mathematically, the equilibrium point is defined as follows:

Definition

*Given an autonomous system*

$$\dot{x} = f(x), \quad x \in R^n$$
………(3)

The *point* $x \in R^n$ is called the equilibrium point of the system (3) if $f(\bar{x}) = 0$

**Eigenvalues and Eigenvectors**

Definition

*Suppose* $A$ *is an* $nxn$ *matrix, then the vector* $x \in C^n, x \neq 0$ *is called the eigenvector of* $A$ *if* $Ax$ *is a scalar multiple of* $x$ *that is*

$$AX = \lambda I$$
………(4)

*for a scalar. The scalar is called the eigenvalue of A and x is called the eigenvector which corresponds to:*

For to be an eigenvalue, there must be one nonzero solution of equation (4). Equation (4) has a nonzero solution if and only if

$$det(\lambda I - A) = 0$$
………(5)

Equation (5) is called the characteristic equation of the matrix A. Scalars which satisfies equation (5) are eigenvalues A (Anton, 1992).

Theorem

Given the Jacobian matrix $Jf(\bar{x})$ of the nonlinear system $\dot{x} = f(x)$ with eigenvalues $\lambda$.

4. Locally asymptotically stable, if all real parts of the eigenvalues of the matrix $Jf(\bar{x})$ negative value.
5. Unstable, if there is at least one eigenvalue matrix $Jf(\bar{x})$ which the real part is positive.

**Routh-Hurwitz Criteria**

Based on theorem 1, to test the stability properties, calculations are needed to determine the eigenvalues of the Jacobian matrix at the equilibrium point. The Routh-Hurwitz criterion is an alternative to determine the eigenvalues. Given a system of characteristic equations in polynomial form as follows:

$$f(s) = a_0 s^n + a_1 s^{n-1} + a_3 s^{n-2} + .. + a_{n-1} s + a_n$$

If equation (6) has a negative real part then

$$\frac{a_1}{a_0} > 0, \frac{a_2}{a_0} > 0, \ldots, \frac{a_n}{a_0} > 0$$

**Basic Reproductive Number**

The basic reproduction number denoted by $R_0$ is a measure of potential spread of disease in a population. The basic reproductive number is defined as the expected number of susceptible populations that become infected during period of infection lasts (Wiraya, 2020). According to Guardiola and Vecchio, the number that expresses the average number of secondary effective individuals due to contracting primary individuals that takes place in the susceptible population is called basic reproduction number. Meanwhile, according to Rost and Wu (2008), the theorem about The basic reproduction number is as follows:

1. Asymptotically stable disease free equilibrium point local if $R_0 < 1$ and unstable if $R_0 > 1$.
2. If $R_0 < 1$ then all solutions converge to a disease-free equilibrium point (disease free equilibrium).
3. The local asymptotic endemic equilibrium point if $R_0 > 1$.
4. If $R_0 > 1$ then the disease is endemic.

**Mathematical Modeling**

A model is a representation of a reality from a modeler or in other words a model is a bridge between the real world and the world of thought(thinking) to solve a problem. The process of elaborating or representing this is referred to as modeling or modeling which is nothing but a process think through logical sequences . Mathematical modeling is a process that undergoes three stages as follows:

1. Formulation of mathematical models.
2. Completion and analysis of mathematical models.
3. Interpreting the results into real intuasi

**Epidemic Model**

The basic model of an epidemic is known as SIR. In this model the population is divided into three sub-populations, namely susceptible sub-populations (susceptibility to infection), infectious (infected) and removed (immune or dead in quarantine) sub-populations . From the basic model, the model can develop into other models. The growth of each sub population will be modeled by a differential equation, so that the growth rate of the sub population will be form a differential system. In this study, the SITR model will be discussed (Murugesan & Subramanian, 2016). Where in this model, the population divided into population groups, namely:

1. Susceptible ($S(t)$): population of humans who are healthy but susceptible to infection with the virus at time t.
2. Infected ($I(t)$): population of humans infected with the virus at time t.
3. Treatment ($T(t)$): population of humans who received treatment at time t.
4. Recovered ($R(t)$): population of humans who have recovered at time t

This study discusses the SITR model on the transmission of influenza virus in the human body by treatment. The model compiled is a mathematical model in the form of a differential equation that depends on the variables that represent each population. After the model is formed, next find the equilibrium point and the basic reproduction ratio ($R_0$)

## RESULT AND DISCUSSION

Simulation of the value of the basic reproduction number ($R_0$) is carried out on the rate of spread of individuals who are susceptible to becoming infected ($\beta$) and the rate of healing of individuals who are susceptible to infection vulnerable ($|Lambda$), so that by substituting an existing parameter value thenobtained a simulation of $R_0(\beta, \Lambda)$ is.

$$R_0(\beta, \Lambda) = \frac{\beta\Lambda}{0{,}453}$$

with $\beta \in [(0.1),(1)]$ and $\Lambda \in [(0.1),(1)]$

For example, when $\beta = 0.1$ and $\Lambda = 0.1$, the value of $R_0$ = 0.02208,then when$\beta = 0.1$ and $\Lambda = 0.2$ then we get $R_0 = 0.04415$ or when $\beta = 1$ and $\Lambda = 1$ then obtained $R_0 = 2.2075$. Therefore, $R_0 = 0.02208$ as the minimum value and $R_0 = 2.2075$ as the maximum value. so that the value of $R_0 = 0.02208$ in this study is called the basic disease-free reproductive number and $R_0 = 2.2075$ is called the endemic basic reproduction number

Simulations are carried out by assigning values for each parameter according to the condition of the value of R0 in the theorems that have been given. The simulation obtained explains that the conditions for it to not occur are endemic is $R_0 < 1$. The value of $R_0 < 1$ is obtained when $\beta \in [(0.1),(0.6)]$ and $\Lambda \in [(0.1),(0.6)]$. While $\beta \in [(0.7),(1)]$ and $\Lambda \in [(0.7),(1)]$ cause $R_0 > 1$ which means that endemic influenza. The following table explains these statements. The following table of simulation $R_0(\beta, \Lambda)$ results:

**Table.** 1 Disease-Free $R_0$ value

| | $\beta$ | $\Lambda$ | $R_0$ | Identify |
|---|---|---|---|---|
| 1 | 0.1 | | 0.02208 | Disease Free |
| 2 | 0.2 | | 0.08830 | Disease Free |
| 3 | 0.3 | | 0.19868 | Disease Free |
| 4 | 0.4 | | 0.35320 | Disease Free |
| 5 | 0.5 | | 0.55188 | Disease Free |
| 6 | 0.6 | | 0.79470 | Disease Free |

**Table.** 2 Endemic $R_0$Value

| $\beta$ | $\Lambda$ | $R_0$ | Identify |
|---|---|---|---|
| 0.7 | 0.7 | 1.08168 | Endemic |
| 0.8 | 0.8 | 1.41280 | Endemic |
| 0.9 | 0.9 | 1.78808 | Endemic |
| 1.0 | 1.0 | 2.20750 | Endemic |

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## CONCLUSION

From the discussion in the previous chapter, it can be concluded The mathematical model for the transmission of influenza virus in the human body with herbal medicine has two equilibrium points, namely disease-free equilibrium point $E_0 = (S, I, T, R) = \left(\frac{\Lambda}{\mu}, 0,0,0\right)$. Where the Jacobian matrix forms a characteristic polynomial, the eigenvalues of 1, 2, 3 and 4 are obtained for each condition of $R_0$. So that the equilibrium point $E_0$ is locally asymptotically stable at time $R_0$. Endemic equilibrium point $E_1 = (S^*, I^*, T^*, R^*)$is.

$$S^* = \frac{\mu + \gamma + \delta + \mu_i}{\beta}$$

$$I^* = \frac{\beta\Lambda - \mu(\mu + \gamma + \delta + \mu_1)}{\beta(\mu + \gamma + \delta + \mu_1)}$$

$$T^* = \frac{\gamma I^*}{\alpha + \mu}$$

$$R^* = \frac{\{\alpha\gamma + \delta(\alpha + \mu)\}I}{\mu(\alpha + \mu)}$$

Where the Jacobian matrix forms a characteristic polynomial and obtains the characteristic equation $q(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda^1 + a_4$. To find out the eigenvalues whose real part is negative from the characteristic equation, we use the Routh-Hurwitz criteria that meet the following conditions:

$$a_1 > 0, a_2 > 0, a_3 > 0$$

$$a_1 . a_2 - a_3 > 0$$

Obviously for $\lambda_2, \lambda_3$ and $\lambda_4$ are negative and if $R_0 > 1$. So the equilibrium point $E_0$ is locally asymptotically stable at time $R_0$. The simulation obtained explains that the conditions for it to not occur are endemic is $R_0 < 1$. The value of $R_0 < 1$ is obtained when $\beta \in [(0.1), (0.6)]$ and $\Lambda \in [(0.1), (0.6)]$. While $\beta \in [(0.7), (1)]$ and $\Lambda \in [(0.7), (1)]$ cause $R_0 > 1$ which means that endemic influenza.

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